March 28, 2014 **MATHEMATICS 218** NAME. -------------------
Time : 55 minutes QUIZ II ID# -----------------
-
Spring 2013-14

Circle your section number :

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1	2	3	4	5	6	7	8	9	10	11	12
9 M	2 F	8 M	1 W	2 F	1 M	3:30 T	5 T	12:30 T	1 F	11 M	11 F

PROBLEM GRADE

PART I

1 ------- /20

2 a. ------- /12

b. ------- /12

3 ------- / 15

4. ------- / 10

PART II

5	**6**	**7**	**8**	**9**
a	a	a	a	a
b	b	b	b	b
c	c	c	c	c
d	d	d	d	d
e	e	e	e	e

5-9 ------- / 15

PART III Answer **True** or **False only** in the table below**:**

A	**B**	**C**	**D**	**E**	**F**	**G**	**H**

10 ------- / 16

TOTAL ------- /100

PART I. Answer each of the following problems in the space provided for each problem (Problem 1 to Problem 4).

1. Let $A=\begin{pmatrix} 1 & -1 & -1 & 1 \\ 2 & -1 & -3 & 4 \\ 3 & -3 & -2 & 6 \\ 1 & -1 & 0 & 4 \end{pmatrix}$

(a) Find a basis of the null space N(A).
(b) Find a basis of the column space Col(A).

[20 points]

2. Show that each of the following is a **subspace** of the corresponding vector space and find a **basis** for each:

(a) Let U be the subset of $\mathbb{R}^3$ defined by:

$$U=\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in R^3 \mid z=x-y \}$$

[12 points]

Basis of U:

2(b) W= { $p(x) \in P_3 : p'(1) = 0$ }.

[12 points]

Basis of W:

3. Show that if $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is a basis for a vector space V, then $\{2\mathbf{u} - \mathbf{v} - \mathbf{w}, 3\mathbf{u} - \mathbf{v}, 2\mathbf{w}\}$ is a basis for V.

[15 points]

4. Let V be a vector space. Let U and W be subspaces of V such that $U \cap W = \{0\}$.
Suppose that $\{\mathbf{u_1}\}$ is a basis of U, and $\{\mathbf{w_1}, \mathbf{w_2}, \mathbf{w_3}\}$ is a basis of W.
Show that the set $\{\mathbf{u_1}, \mathbf{w_1}, \mathbf{w_2}, \mathbf{w_3}\}$ **is linearly independent.**

[10 points]

PART II. Circle the correct answer for each of the following problems (Problem 5 to Problem 9) IN THE TABLE OF THE FRONT PAGE . [3 points for each correct answer].

5. The subspace of R^3 spanned by $\left\{ \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix} \right\}$ has a dimension equal to

(a) 1
(b) 2
(c) 4
(d) 3
(e) none of the above.

[3 points]

6. Let V be a vector space of dimension n. Which one of the following statements is FALSE:

a. Any set of (n+1) vectors in V is linearly dependent.
b. Any linearly independent set of n vectors in V is a basis of V.
c. Any set of n vectors spanning V is a basis of V.
d. Any set of n vectors in V spans V.
e. none of the above.

[3 points]

7. Let V= $\{ p(x) \in P_3 : p(1) = 0 \text{ and } p(0) = 0 \}$. *Then* dim V=

(a) 1
(b) 2
(c) 3
(d) 4
(e) none of the above.

[3 points]

8. Let S be the space of all upper triangular 3×3 matrices A such that the sum of all entries of A is zero. Then dim S is equal to:

(a) 3
(b) 6
(c) 4
(d) 5
(e) none of the above.

[3 points]

9. Let $T : \mathbf{R}^2 \to \mathbf{R}$ be a linear transformation such that $T\begin{pmatrix}1\\0\end{pmatrix} = 8$ *and* $T\begin{pmatrix}1\\1\end{pmatrix} = 3$, *then* $T\begin{pmatrix}5\\9\end{pmatrix} =$

(a) 6
(b) -6
(c) 5
(d) -5
(e) none of the above.

[3 points]

10. Answer **TRUE** or **FALSE** only (2 points for each correct answer) **IN THE TABLE IN THE FRONT PAGE**

A. Any set of 2 vectors in R^3 can be extended (enlarged) to become a basis of R^3.

B. The polynomials $2+3x$, $3-4x^3$, $x+x^3$, $1+x-x^2$, $5x+2x^2$ are linearly dependent . in P_3

C. Any subspace of a vector space is linearly independent.

D. Let $W=\left\{\begin{pmatrix}a & 0\\ b & 1\end{pmatrix} \in M_{2\times 2} \mid \ a, b \in R\right\}$, then W is a subspace of $M_2 \times_2$.

E. The space of all symmetric 3×3 matrices has dimension 5.

F. Let W be a subspace of a vector space V, then the set $U = \{A \in V : A \notin W\}$ is a subspace V.

G. If $T:V \to W$ is a linear transformation and if $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ is a linearly independent subset of V, then $\{T(\mathbf{u_1}), T(\mathbf{u_2}), T(\mathbf{u_3})\}$ is linearly independent in W.

H. Let $V=\{ p(x) \in P_2 : p''(x) = 0\}$. Then dim V=2.

[16 points]